Jackson 5.21 (a)

We are interested in field due entirely to localized distribution of permanent magnetization, this implies no current.

$\Rightarrow$ $\vec{\nabla} \times \vec{H} = \vec{J}_f = 0$, so ~~Fet~~ we must have $\vec{H} = \vec{\nabla} \Phi$ for some Φ.

$$\Rightarrow \quad \int \vec{B} \cdot \vec{H} \, d^3x = \int \vec{B} \cdot (\vec{\nabla} \Phi) \, d^3x .$$

vector identity: $\vec{\nabla}(\psi \vec{A}) = (\vec{\nabla}\psi) \cdot \vec{A} + \psi(\vec{\nabla} \cdot \vec{A})$

$$\Rightarrow \quad \vec{B} \cdot (\vec{\nabla} \Phi) = \vec{\nabla}(\Phi \vec{B}) - \Phi(\vec{\nabla} \cdot \vec{B})$$

$$\int \vec{B} \cdot \vec{H} \, d^3x = \int \left[\vec{\nabla}(\Phi \vec{B}) - \Phi(\vec{\nabla} \cdot \vec{B}) \right] d^3x$$

$$= \Phi \vec{B} \Big|_{-\infty}^{\infty} - \int \Phi (\vec{\nabla} \cdot \vec{B})$$

vanishes because field is localized

vanishes because $\vec{\nabla} \cdot \vec{B} = 0$ always.

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Jackson 5.21 (b)

The energy of a permanent magnetic moment (dipole) in magnetic field $\vec{B}$ is given by (5.72):

$$U = -\vec{m} \cdot \vec{B}.$$

Switching to continuous distribution, it is

$$U = -\int \frac{\vec{M}}{2} \cdot \vec{B} \, d^3x$$

where $\vec{M}$ is the magnetic dipole density, and the factor of 2 is to factor out double counting.

Invoking $\vec{B} = \mu_0(\vec{H} + \vec{M})$, we have

$$U = -\int \frac{\vec{M}}{2} \cdot \mu_0(\vec{H}+\vec{M}) = -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}\, d^3x - \frac{\mu_0}{2}\int \vec{M}\cdot\vec{M}\, d^3x$$

$$= -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}\, d^3x - \frac{\mu_0}{2}\int |\vec{M}|^2\, d^3x$$

This term is translationally and rotationally invariant, and only depends on the total amount of permanent magnets in ~~the~~ all space

$$\Rightarrow \boxed{U = -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}\, d^3x - c} \qquad c = \frac{\mu_0}{2}\int |\vec{M}|^2\, d^3x$$

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